

Intermediate inflation from a non-canonical scalar field

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We study the intermediate inflation in a non-canonical scalar field framework with a power-like Lagrangian. We show that in contrast with the standard canonical intermediate inflation, our non-canonical model is compatible with the observational results of Planck 2015. Also, we estimate the equilateral non-Gaussianity parameter which is in well agreement with the prediction of Planck 2015. Then, we obtain an approximation for the energy scale at the initial time of inflation and show that it can be of order of the Planck energy scale, i.e. $M_P \sim 10^{18}$ GeV. We will see that after a short period of time, inflation enters in the slow-roll regime that its energy scale is of order $M_P/100 \sim 10^{16}$ GeV and the horizon exit takes place in this energy scale. We also examine an idea in our non-canonical model to overcome the central drawback of intermediate inflation which is the fact that inflation never ends. We solve this problem without disturbing significantly the nature of the intermediate inflation until the time of horizon exit.

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I. INTRODUCTION

Inflationary scenario is one important part of modern cosmology. In this scenario, it is believed that a rapid expansion has occurred in the very early stages of our universe. Consideration of this fast accelerated expansion can resolve some of basic problems of the Hot Big Bang cosmology, such as horizon problem, flatness problem and relic particle abundances problem [1–7] (see also [8–13] for reviews on inflation). One important consequence of the inflationary paradigm is the fact that the growth of perturbation generated during inflation can provide a convincing explanation for the Large Scale Structure (LSS) formation in the universe and also for the observed anisotropy in the Cosmic Microwave Background (CMB) radiation [14–17] (see also [18–22] for reviews on cosmological perturbations theory). Inflation generates two types of perturbations, namely, the scalar perturbations and the tensor perturbations. The scalar perturbations are responsible for the density perturbations while the tensor fluctuations lead to the gravitational waves [18–22]. Inflationary scenario predicts a nearly scale invariant, adiabatic and Gaussian spectrum for the scalar perturbations [8]. These predictions are confirmed with the experimental results from exploring the anisotropy in the CMB temperature angular power spectrum by Planck satellite [23]. So far, the accurate data from exploring CMB spectrum has narrowed the range of acceptable inflationary models [24–26]. Furthermore, increasingly accurate measurements in the future will discriminate more tightly between the inflationary models and will provide us with more information about inflation dynamics.

The standard inflationary model is based on a single scalar field called “inflaton”, and a potential which determines the evolution of this field during inflation [8–13]. In the standard model of inflation, a canonical kinetic term is included in Lagrangian and usually this term is dominated by the potential term. But also there are some models of inflation in which the kinetic term can be different from the standard canonical one [27–37]. These models are known as the non-canonical models of inflation. One important class of non-canonical models is k -inflation in which the kinetic term can dominate the potential one [27, 28]. Perturbations in k -inflation and observational constraints on this model have been studied in [28] and [29], respectively.

Furthermore, some other considerable works have been done in the framework of non-canonical inflationary scenario [30–37]. For instance, in [30] a non-minimal term coupled to

the gravity action was considered and consequently the equations governing the inflationary observables were derived. Some viable Lagrangians for non-canonical inflation were studied in [31] and their attractor behavior in phase space was examined. Moreover, in [31] some conditions for Lagrangian of non-canonical inflation were expressed in order to hold the null energy condition as well as the condition of physical propagation of perturbations. A detailed discussion about the initial conditions in phase space for non-canonical inflation was also represented in [32]. In [33, 34], the authors tried to refine the well-known inflationary models in light of observational results in the framework of non-canonical scenario. In [35], the non-canonical inflation was also extended to the warm inflationary scenario in which the radiation is produced during inflation continuously so that one recovers the radiation dominated era without need to any reheating process.

It has also been shown that by use of a non-canonical Lagrangian, we can reduce the values of slow-roll parameters and consequently, the condition of the slow-roll regime can be reached more easily relative to the canonical case [33]. In this way, we can increase the scalar spectral index while decreasing the tensor-to-scalar ratio [33]. Consequently, such models as the quartic potential $V(\phi) = \frac{1}{4}\lambda\phi^4$ which has self interaction and the quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ can be made more compatible with the observational results relative to the standard canonical inflation [33]. Also, it has been clarified that the steep potentials including the inverse power law potential and the exponential potential, which are associated with dark energy in the canonical setting, can provide inflation in the non-canonical framework [33]. It has also been pointed out that in the non-canonical setting, we can resolve the problems of the power law inflation [34]. In other words, in the non-canonical setting, the power law inflation can be compatible with the observational results and we can provide a way for the inflation to end without changing significantly the power law form of the scale factor around the horizon exit [34].

Here, we focus on the intermediate inflation with a scale factor in the form of $a(t) \propto \exp(At^f)$ where $A > 0$ and $0 < f < 1$ [38–40]. The expansion of the universe with this scale factor is slower than the de Sitter inflation ($a(t) \propto \exp(Ht)$ where H is constant), but faster than the power law inflation ($a(t) \propto t^q$ where $q > 1$). The intermediate inflation has already been studied in the framework of standard model of inflation [38–40]. It was shown that the intermediate inflation arises as the slow-roll solution to potentials which fall off asymptotically as an inverse power law inflation in the standard canonical framework and

can be modelled by an exact cosmological solution [39, 40]. The intermediate inflation has also been studied in some warm inflationary scenarios in order to examine its predictions for inflationary observables [41–43].

The intermediate inflation suffers from some problems in the standard canonical inflation scenario. In [39], it was shown that the intermediate inflation represents the scalar and tensor power spectra which are disfavored in light of the observational results from COBE satellite. Also, the intermediate inflation never goes to an end without invoking any additional process [39]. In the present paper, our main goal is to refine these problems by considering intermediate inflation in a non-canonical framework.

This paper is organized as follows. In section II, we consider the intermediate inflation from a non-canonical scalar field and estimate the inflationary observables and compare them with the results of Planck 2015. Then, we find an estimation for the energy scale at the beginning of inflation in our model. In section III, we investigate how the graceful exit problem can be solved in our non-canonical model. Section IV is devoted to conclusions.

II. INTERMEDIATE INFLATION IN A NON-CANONICAL FRAMEWORK

Let us consider the following action

$$S = \int d^4x \sqrt{-g} \mathcal{L}(X, \phi), \quad (1)$$

where \mathcal{L} , ϕ and $X \equiv \partial_\mu \phi \partial^\mu \phi / 2$ are the Lagrangian, the inflaton scalar field and the kinetic term, respectively. The energy density ρ_ϕ and pressure p_ϕ of the scalar field for the above action are given by [27–35]

$$\rho_\phi = 2X \left(\frac{\partial \mathcal{L}}{\partial X} \right) - \mathcal{L}, \quad (2)$$

$$p_\phi = \mathcal{L}. \quad (3)$$

The equation of state parameter is defined as

$$\omega_\phi \equiv \frac{p_\phi}{\rho_\phi}. \quad (4)$$

We consider the Friedmann-Robertson-Walker (FRW) metric for a flat universe,

$$ds^2 = dt^2 - a^2(t) (dx^2 + dy^2 + dz^2), \quad (5)$$

where $a(t)$ is scale factor of the universe. For the above metric, the kinetic term turns into $X = \dot{\phi}^2/2$. Dynamics of the universe for the flat FRW metric in the Einstein gravity is determined by the Friedmann equation

$$H^2 = \frac{1}{3M_P^2} \rho_\phi, \quad (6)$$

together with the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2} (\rho_\phi + 3p_\phi), \quad (7)$$

where $M_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass and $H \equiv \dot{a}/a$ is the Hubble parameter. The energy density of the inflaton scalar field, ρ_ϕ , satisfies the conservation equation

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0. \quad (8)$$

The first and second slow-roll parameters are defined as

$$\varepsilon = -\frac{\dot{H}}{H^2}, \quad (9)$$

$$\eta = \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}, \quad (10)$$

respectively. From the definition of ε , we reach the condition $\varepsilon < 1$ to have inflation ($\ddot{a} > 0$). We know that the Hubble parameter is approximately constant during inflation and also the accelerated expansion should be sustained for a sufficiently long period of time. Hence, we should have $\varepsilon \ll 1$ and $|\eta| \ll 1$ and the assumption of these conditions is known as the slow-roll approximation.

It is convenient to express the amount of inflation with respect to the e -fold number defined as

$$N \equiv \ln \left(\frac{a_e}{a} \right), \quad (11)$$

where a_e is the scale factor at the end of inflation. The above definition leads to

$$dN = -Hdt = -\frac{H}{\dot{\phi}} d\phi. \quad (12)$$

In order to solve the problems of Hot Big Bang cosmology, we need more than 60 e -folds [44].

In this paper, we assume that in the action (1), the Lagrangian has the power-like form

$$\mathcal{L}(X, \phi) = X \left(\frac{X}{M^4} \right)^{\alpha-1} - V(\phi), \quad (13)$$

where α is a dimensionless parameter and M is a parameter with dimensions of mass [33, 34]. For $\alpha = 1$, the above Lagrangian turns into the standard canonical Lagrangian $\mathcal{L}(X, \phi) = X - V(\phi)$. Therefore, we can consider the Lagrangian (13) as a generalized form of the standard canonical Lagrangian. This Lagrangian satisfies the conditions $\partial\mathcal{L}/\partial X \geq 0$ and $\partial^2\mathcal{L}/\partial X^2 > 0$ required for the null-energy condition and the condition of physical propagations of perturbations, respectively [31]. This Lagrangian has been considered before to refine some chaotic inflationary models and steep potentials [33], and also to resurrect the power law inflation in light of Planck 2013 results as well as to suggest a reasonable idea for the end of power law inflation [34].

Inserting the Lagrangian (13) into Eqs. (2) and (3), we find the energy density and pressure of the scalar field ϕ as

$$\rho_\phi = (2\alpha - 1) X \left(\frac{X}{M^4} \right)^{\alpha-1} + V(\phi), \quad (14)$$

$$p_\phi = X \left(\frac{X}{M^4} \right)^{\alpha-1} - V(\phi). \quad (15)$$

Using the above relations in the conservation equation (8) leads to the evolution equation of the scalar field as

$$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} + \left(\frac{V'(\phi)}{\alpha(2\alpha - 1)} \right) \left(\frac{2M^4}{\dot{\phi}^2} \right)^{\alpha-1} = 0. \quad (16)$$

We can show that by use of the slow-roll conditions for the Lagrangian (13), the first and second slow-roll parameters, (9) and (10), are related to the potential $V(\phi)$ as

$$\varepsilon_V = \left[\frac{1}{\alpha} \left(\frac{3M^4}{V(\phi)} \right)^{\alpha-1} \left(\frac{M_P V'(\phi)}{\sqrt{2} V(\phi)} \right)^{2\alpha} \right]^{\frac{1}{2\alpha-1}}, \quad (17)$$

$$\eta_V = \left(\frac{\alpha\varepsilon_V}{2\alpha - 1} \right) \left(\frac{2V(\phi)V''(\phi)}{V'(\phi)^2} - 1 \right). \quad (18)$$

The above quantities, are called the first and second potential slow-roll parameters, respectively. Also, in the slow-roll regime the potential energy dominates the kinetic energy and thus the Friedmann equation (6) reduces to

$$H^2(\phi) = \frac{1}{3M_P^2} V(\phi). \quad (19)$$

Moreover, in the slow-roll regime, the evolution equation of the scalar field, (16), takes the form

$$\dot{\phi} = -\theta \left\{ \left(\frac{M_P}{\sqrt{3}\alpha} \right) \left(\frac{\theta V'(\phi)}{\sqrt{V(\phi)}} \right) (2M^4)^{\alpha-1} \right\}^{\frac{1}{2\alpha-1}}, \quad (20)$$

where $\theta = 1$ when $V'(\phi) > 0$ and $\theta = -1$ when $V'(\phi) < 0$.

In this paper, we are interested in studying the intermediate inflation with the scale factor

$$a(t) = a_i \exp \left[A(M_P t)^f \right], \quad (21)$$

where $A > 0$ and $0 < f < 1$ [38–40]. a_i is the scale factor at the initial time of inflation. Throughout this paper, we normalize the scale factor to its value at the present time, $a_0 = 1$. The reduced Planck mass M_P was applied to make the argument of the exponential function be dimensionless.

With the help of Eqs. (6) and (7) for the intermediate scale factor (21), we find

$$\rho_\phi = 3A^2 f^2 (M_P t)^{2f-2} M_P^4, \quad (22)$$

$$p_\phi = -A f (M_P t)^{f-2} \left[f \left(3A (M_P t)^f + 2 \right) - 2 \right] M_P^4. \quad (23)$$

Equating (14) and (22) and also using $X = \dot{\phi}^2/2$, we obtain

$$V(t) = 3A^2 f^2 (M_P t)^{2f-2} M_P^4 - 2^{-\alpha} (2\alpha - 1) M^{-4(\alpha-1)} \dot{\phi}^{2\alpha}. \quad (24)$$

Inserting Eq. (24) into (15) and equating the obtained result with Eq. (23), we reach a differential equation which its solution reads

$$\phi(t) = \frac{2\sqrt{2}\alpha^{\frac{2\alpha-1}{2\alpha}} \bar{M}^{\frac{2(\alpha-1)}{\alpha}} (A f (1-f))^{\frac{1}{2\alpha}} (M_P t)^{\frac{2\alpha+f-2}{2\alpha}}}{2\alpha + f - 2} M_P + \phi_0, \quad (25)$$

where $\bar{M} \equiv M/M_P$ and ϕ_0 is the constant of integration that we take it as $\phi_0 = 0$ without loss of generality. Now, we use the above solution in Eq. (24) and get

$$V(t) = \alpha^{-1} A f (M_P t)^{f-2} \left[3\alpha A f (M_P t)^f + 2\alpha (f-1) - f + 1 \right] M_P^4. \quad (26)$$

The above result is exact since we have not applied the slow-roll approximation in its derivation. We will apply the above equation at the end of this section to estimate the energy scale at the start of inflation. In the slow-roll approximation, using the Friedmann equation (19) for the intermediate scale factor (21), we obtain

$$V(t) = 3A^2 f^2 (M_P t)^{2f-2} M_P^4. \quad (27)$$

With the help of Eqs. (25) and (27), we find the form of the inflationary potential in terms of ϕ as

$$V(\phi) = V_0 \left(\frac{\phi}{M_P} \right)^{-s}, \quad (28)$$

where

$$s = \frac{4\alpha(1-f)}{2\alpha+f-2}, \quad (29)$$

and

$$V_0 = \frac{3 \times 2^{\frac{6\alpha(1-f)}{2\alpha+f-2}} \alpha^{\frac{2(2\alpha-1)(1-f)}{2\alpha+f-2}} \bar{M}^{\frac{8(\alpha-1)(1-f)}{2\alpha+f-2}} (Af)^{\frac{4\alpha-2}{2\alpha+f-2}} (1-f)^{\frac{2(1-f)}{2\alpha+f-2}}}{(2\alpha+f-2)^{\frac{4\alpha(1-f)}{2\alpha+f-2}}} M_P^4. \quad (30)$$

We see that the potential driving the intermediate inflation in our non-canonical framework, like the potential of the standard canonical case [40], has an inverse power law form. Since the value of f for the intermediate scale factor (21) should be between 0 and 1, from Eq. (29) we conclude that for a given value of α , the parameter s in the potential (28) must be in the range $0 < s < 2\alpha/(\alpha-1)$ to have intermediate inflation in the non-canonical setting whereas in the standard canonical setting ($\alpha = 1$), the parameter s can take any positive value.

Having the inflationary potential, we can obtain the relations needed for calculating the inflationary observables. In the slow-roll approximation, the power spectrum of scalar perturbations for our non-canonical model (13) acquires the form [33, 34]

$$\mathcal{P}_s = \frac{1}{72\pi^2 c_s} \left(\frac{6^\alpha \alpha V(\phi)^{5\alpha-2}}{M_P^{14\alpha-8} \bar{M}^{4(\alpha-1)} V'(\phi)^{2\alpha}} \right)_{aH=c_s k}^{\frac{1}{2\alpha-1}}. \quad (31)$$

This quantity should be evaluated at the sound horizon exit specified by $aH = c_s k$ where k is the comoving wavenumber and c_s is the sound speed defined as [27–35]

$$c_s^2 \equiv \frac{\partial p_\phi / \partial X}{\partial \rho_\phi / \partial X} = \frac{\partial \mathcal{L}(X, \phi) / \partial X}{(2X) \partial^2 \mathcal{L}(X, \phi) / \partial X^2 + \partial \mathcal{L}(X, \phi) / \partial X}. \quad (32)$$

For our non-canonical model (13), it reduces to

$$c_s = \frac{1}{\sqrt{2\alpha-1}}, \quad (33)$$

which is a constant quantity.

Substituting the potential (28) into Eq. (31) and after some simplifications, we get

$$\mathcal{P}_s = \frac{\sqrt{2\alpha-1} (Af)^{\frac{6\alpha-4}{2\alpha+f-2}} (2\alpha+f-2)^{\frac{\alpha(6f-4)}{2\alpha+f-2}}}{2^{\frac{3(3\alpha f+f-2)}{2\alpha+f-2}} \pi^2 \alpha^{\frac{(2\alpha-1)(3f-2)}{2\alpha+f-2}} \bar{M}^{\frac{4(\alpha-1)(3f-2)}{2\alpha+f-2}} (1-f)^{\frac{2(\alpha+2f-2)}{2\alpha+f-2}}} \left(\frac{\phi}{M_P} \right)_{aH=c_s k}^{\frac{\alpha(6f-4)}{2\alpha+f-2}}. \quad (34)$$

In the above equation, we see that for the value of $f = 2/3$, the scalar power spectrum is independent of the scalar field ϕ and we expect a scale-invariant Harrison-Zel'dovich spectrum. Now, we use Eq. (25) in (34) and after some simplifications, we obtain

$$\mathcal{P}_s = \frac{\sqrt{2\alpha-1} A^3 f^3}{8\pi^2 (1-f)} (M_P t)_{aH=c_s k}^{3f-2}. \quad (35)$$

Here, we solve the equation $aH = c_s k$ and get the time of sound horizon exit as

$$t_{*s} = \frac{1}{M_P} \left\{ \frac{f-1}{Af} W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{1/f}, \quad (36)$$

where we have used the Lambert W function defined as solution of the equation $ye^y = x$ [45]. In the complex plane, the equation $ye^y = x$ has a countably infinite number of solutions that they are represented by $W_k(x)$ with k ranging over the integers. For all real $x \geq 0$, the equation has exactly one real solution. It is represented by $y = W(x)$ or, equivalently, $y = W_0(x)$. For all real x in the range $x < 0$, there are exactly two real solutions. The larger one is represented by $y = W(x)$ and the smaller one is denoted by $y = W_{-1}(x)$.

With the help of Eqs. (35) and (36), we find the scalar power spectrum in terms of the comoving wavenumber k as

$$\mathcal{P}_s(k) = \frac{A^3 f^3}{8\pi^2 c_s (1-f)} \left\{ \frac{f-1}{Af} W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{\frac{3f-2}{f}}. \quad (37)$$

The scalar spectral index is defined as

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_s}{d \ln k}. \quad (38)$$

Therefore, using Eq. (37), we obtain

$$n_s = 1 + \frac{3f-2}{f-1} \left\{ W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] + 1 \right\}^{-1}. \quad (39)$$

In our model, we also include the running of the scalar spectral index given by

$$\frac{dn_s}{d \ln k} = \frac{f(2-3f) W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right]}{(f-1)^2 \left\{ W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] + 1 \right\}^3}. \quad (40)$$

The power spectrum of the tensor perturbations for our non-canonical model (13) is the same as one for the standard canonical model and is given by [28]

$$\mathcal{P}_t = \frac{2}{3\pi^2} \left(\frac{V(\phi)}{M_P^4} \right)_{aH=k}, \quad (41)$$

where it should be calculated at the horizon exit specified by $aH = k$. Since the above equation is unaffected by the value of α in the Lagrangian (13), the energy scale at the horizon exit is same in both canonical and non-canonical models.

Inserting the potential (28) into Eq. (41), we obtain

$$\mathcal{P}_t = \frac{2^{\frac{8\alpha-6\alpha f+f-2}{2\alpha+f-2}} \alpha^{\frac{2(2\alpha-1)(1-f)}{2\alpha+f-2}} M^{-\frac{8(\alpha-1)(1-f)}{2\alpha+f-2}} (Af)^{\frac{4\alpha-2}{2\alpha+f-2}} (1-f)^{\frac{2(1-f)}{2\alpha+f-2}} \left(\frac{\phi}{M_P}\right)_{aH=k}^{-\frac{4\alpha(1-f)}{2\alpha+f-2}}}{\pi^2(2\alpha+f-2)^{\frac{4\alpha(1-f)}{2\alpha+f-2}}} \quad (42)$$

Substituting $\phi(t)$ from Eq. (25) into the above equation leads to

$$\mathcal{P}_t = \frac{2A^2 f^2}{\pi^2} (M_{Pt})_{aH=k}^{-2(1-f)}. \quad (43)$$

Solving the equation $aH = k$, we get the time of horizon exit as

$$t_* = \frac{1}{M_P} \left\{ \frac{f-1}{Af} W_{-1} \left[\frac{Af}{f-1} \left(\frac{k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{1/f}, \quad (44)$$

that can also be obtained by setting $c_s = 1$ in Eq. (36). Now, we use Eq. (44) in (43) and obtain the tensor power spectrum in terms of the comoving wavenumber k as

$$\mathcal{P}_t(k) = \frac{2A^2 f^2}{\pi^2} \left\{ \frac{f-1}{Af} W_{-1} \left[\frac{Af}{f-1} \left(\frac{k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{-\frac{2(1-f)}{f}}. \quad (45)$$

The tensor spectral index is defined as

$$n_t \equiv \frac{d \ln \mathcal{P}_t}{d \ln k}. \quad (46)$$

This with the help of Eq. (45) yields

$$n_t = 2 \left\{ W_{-1} \left[\frac{Af}{f-1} \left(\frac{k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] + 1 \right\}^{-1}. \quad (47)$$

An important inflationary observable is the tensor-to-scalar ratio defined as

$$r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_s}, \quad (48)$$

that can simply be obtained by using of Eqs. (45) and (37). Therefore, we find

$$r = \frac{16c_s \left\{ -W_{-1} \left[\frac{Af}{f-1} \left(\frac{c_s k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{\frac{2-3f}{f}}}{\left\{ -W_{-1} \left[\frac{Af}{f-1} \left(\frac{k}{a_i Af M_P} \right)^{\frac{f}{f-1}} \right] \right\}^{\frac{2(1-f)}{f}}}. \quad (49)$$

Inflationary observables are not completely independent and usually there is a consistency relation between them. For an inflation model with the non-canonical Lagrangian (13), the consistency relation is [33, 34]

$$r \approx -8c_s n_t. \quad (50)$$

The above relation is an approximation since the freeze-out epoch for the scalar perturbations is different from the one for the tensor perturbations. We see that the consistency relation for our non-canonical model is different from the standard canonical case where $r = -8n_t$.

So far, we have obtained the relations corresponding to the inflationary observables in terms of the comoving wavenumber. Here, we check the viability of our model in light of the observational results from Planck 2015. We calculate the inflationary observables at the pivot scale $k_0 = 0.05 \text{ Mpc}^{-1}$. We fix the scalar power spectrum in Eq. (37) at the pivot scale as $\mathcal{P}_s(k_0) = 2.207 \times 10^{-9}$ from Planck 2015 TT,TE,EE+lowP data combination [23]. In this way, we find an equation that gives a value for the parameter a_i for each set of the parameters α , A and f . So, we can plot the $r - n_s$ diagram for our model by use of Eqs. (39) and (49). This diagram is shown in Fig. 1 and also the marginalized joint regions 68% and 95% CL allowed by Planck 2015 data are demonstrated in the figure. Predictions of our model are specified by black lines for specified values of α , A and f . In the figure, we see that the standard canonical intermediate inflation ($\alpha = 1$) is disfavored in light of Planck 2015 results. But if we choose α large enough then result of our non-canonical intermediate inflationary model can be lied inside the regions favored according to Planck 2015 data. For instance, if we take $\alpha = 16$, prediction of our model can lie inside the region 68% CL for Planck 2015 TT,TE,EE+lowP data [23]. Also, from the lines with different values of f , we see that for $\alpha = 16$ and $A = 4$, if we consider $0.244 \lesssim f \lesssim 0.272$ then prediction of our model lies inside the joint region 95% CL for Planck 2015 TT,TE,EE+lowP data [23]. It should be noted that as the parameter f approaches $2/3$, the scalar power spectrum goes toward the scale-invariant Harrison-Zel'dovich spectrum ($n_s = 1$) which is not consistent with the Planck 2015 results [23]. For the values of f in the range $2/3 < f < 1$, we will have a blue-tilted spectrum ($n_s > 1$) which is ruled out by the Planck 2015 data [23].

Now, we test the prediction of our model in the $dn_s/d \ln k - n_s$ plane in comparison with the observational results of Planck 2015. For this purpose, we consider $\alpha = 16$ and $A = 4$. Then we use Eqs. (39) and (40) to plot $dn_s/d \ln k$ versus n_s . This plot is shown in Fig. 2 and we see that the prediction of our model can lie insides the joint 68% CL region of Planck 2015 TT,TE,EE+lowP data [23].

In the following, we proceed to estimate the inflationary observables in our model, explicitly. We choose $\alpha = 16$, $A = 4$ and $f = 255/1000$. Thus, Eqs. (29) and (30) give $s = 9536/6051$ and $V_0 = 5.067 \bar{M}^{5960/2017} M_P^4$, respectively. Furthermore, we fix

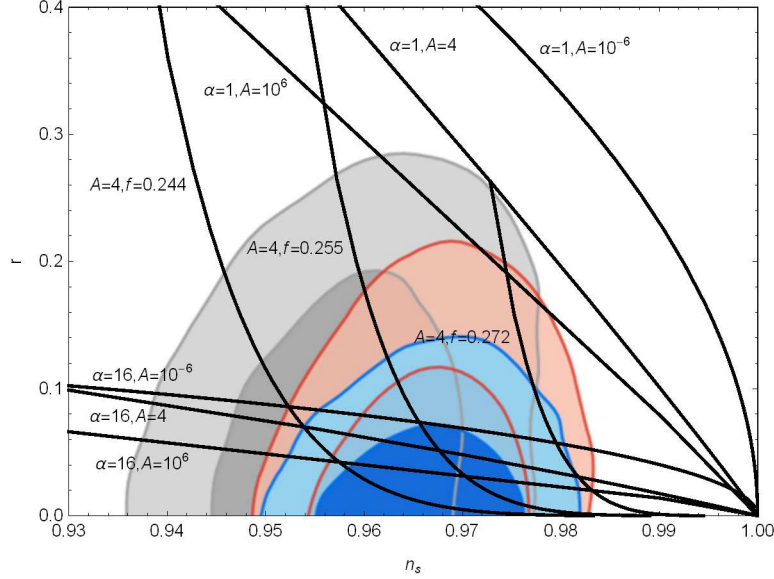


FIG. 1: Prediction of our non-canonical intermediate inflationary model in $r - n_s$ plane in comparison with the observational results of Planck 2015. The thick black lines indicate the predictions of our non-canonical intermediate inflationary model for specified values of α , A and f . The grey, red and blue marginalized joint regions 68% and 95% CL correspond to Planck 2013, Planck 2015 TT+lowP and Planck 2015 TT,TE,EE+lowP data [23], respectively.

$\mathcal{P}_s(k_0) = 2.207 \times 10^{-9}$ from Planck 2015 TT,TE,EE+lowP data [23] in Eq. (37) and determine $a_i = 5.6 \times 10^{-121}$. Now, we can use Eq. (39) and get the scalar spectral index as $n_s = 0.9676$ which lies in the range with 68% CL allowed by Planck 2015 TT,TE,EE+lowP data ($n_s = 0.9644 \pm 0.0049$) [23]. From Eq. (49), we obtain the prediction of our model for the tensor-to-scalar ratio as $r = 0.052$ which is inside the range with 68% CL predicted by Planck 2015 TT,TE,EE+lowP data [23] (see Fig. 1). Furthermore, using Eq. (40), we obtain the running of the scalar spectral index as $dn_s/d \ln k = 0.0002$ which is in agreement with Planck 2015 TT,TE,EE+lowP data at 68% CL [23] (see Fig. 2). From Eq. (47), we see that our model gives the tensor spectral index as $n_t = -0.039$ that can be checked by more accurate measurements in the future. In the consistency relation (50), if we consider the upper bound $r < 0.149$ at 95% CL from Planck TT,TE,EE+lowP [23] and use the sound speed from Eq. (33), we find the constraint $n_t > -0.104$ for the tensor spectral index, which is satisfied by the prediction of our model.

We can estimate the non-Gaussianity parameter in our intermediate non-canonical in-

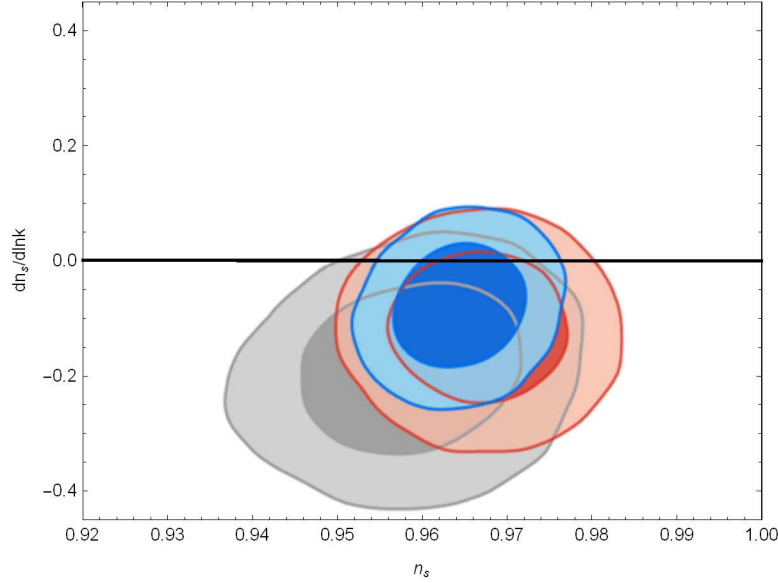


FIG. 2: Prediction of our non-canonical intermediate inflationary model in the $dn_s/d\ln k - n_s$ plane in comparison with the observational results of Planck 2015. The prediction of our model with $\alpha = 16$ and $A = 4$ is shown by a thick black line. The grey, red and blue marginalized joint regions 68% and 95% CL correspond to Planck 2013, Planck 2015 TT+lowP and Planck 2015 TT,TE,EE+lowP data [23], respectively.

flationary model. Non-Gaussianity parameter is another important inflationary observable that can discriminate between inflationary models and it can provide us with some information about the dynamics of scalar field during inflation. For single field inflationary models, the non-Gaussianity parameter has peak in the equilateral shape [12]. Also, if the non-Gaussianity parameter has peak on the squeezed shape, then we conclude that we have multifields inflation [12]. Furthermore, the orthogonal non-Gaussianity arises in models with non-standard initial states [12]. Since in the present work, we deal with a single field inflation, thus we examine the non-Gaussianity parameter in the equilateral limit. For a non-canonical model with the Lagrangian $\mathcal{L}(X, \phi)$, the equilateral non-Gaussianity parameter is given by [49]

$$f_{\text{NL}}^{\text{equil}} = \frac{5}{81} \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} \right) - \frac{35}{108} \left(\frac{1}{c_s^2} - 1 \right), \quad (51)$$

where

$$\lambda = X^2 \frac{\partial^2 \mathcal{L}}{\partial X^2} + \frac{2}{3} X^3 \frac{\partial^3 \mathcal{L}}{\partial X^3}, \quad (52)$$

$$\Sigma = X \frac{\partial \mathcal{L}}{\partial X} + 2X^2 \frac{\partial^2 \mathcal{L}}{\partial X^2}, \quad (53)$$

and the sound speed c_s is given by Eq. (33). Using the above equations for our non-canonical model (13), we get

$$\frac{\lambda}{\Sigma} = \frac{\alpha - 1}{3}. \quad (54)$$

Substituting this together with Eq. (33) into (51) leads to

$$f_{\text{NL}}^{\text{equil}} = -\frac{275}{486}(\alpha - 1), \quad (55)$$

which for $\alpha = 16$ gives $f_{\text{NL}}^{\text{equil}} = -8.5$. This result is in agreement with Planck 2015 results, $f_{\text{NL}}^{\text{equil}} = -16 \pm 70$ at 68% CL [23]. Furthermore, we can easily show that the Planck 2015 bounds on $f_{\text{NL}}^{\text{equil}}$ effectively translate into $1 \leq \alpha \leq 153$ for our non-canonical model.

At the end of this section, we want to obtain an approximation for the energy scale at the start of inflation. To do so, we use Eqs. (9) and (26) that are obtained without applying the slow-roll approximation. Therefore, violation of the slow-roll conditions at the initial times of inflation doesn't disturb the validity of our discussion. We first use the first slow-roll parameter (9) for the intermediate scale factor (21) and reach

$$\varepsilon = \frac{(1 - f)}{Af(M_{Pt})^f}, \quad (56)$$

that is a decreasing function during inflation and hence the equation $\varepsilon = 1$ is related to the initial time of inflation [50]. Therefore, we obtain the initial time of inflation as

$$\bar{t}_i \equiv M_{Pt_i} = \left(\frac{1 - f}{Af} \right)^{1/f}, \quad (57)$$

where $\bar{t} \equiv M_{Pt}$ is dimensionless time. Substituting this into Eq. (26), we get the potential energy at the initial time of inflation as

$$V_i \equiv V(t_i) = \frac{(\alpha + 1)}{\alpha} (Af)^{2/f} (1 - f)^{-2(1-f)/f} M_P^4. \quad (58)$$

We take $\alpha = 16$, $A = 4$ and $f = 255/1000$ as determined above. From Eq. (57), we obtain the initial time of inflation as $\bar{t}_i = 0.29$ or equivalently $t_i = 7.9 \times 10^{-44}$ sec. Also, from Eq. (58), we find the potential energy at the initial time of inflation as $V_i = 6.9 M_P^4$. Therefore,

we obtain the energy scale at the start of inflation as $V_i^{1/4} = 1.6M_P \sim 10^{18}\text{GeV}$ which is of order of the Planck energy scale. Therefore, we can provide a reasonable explanation for one of the mysteries of the inflation theory that the energy scale defined by the energy density of the universe at horizon exit is a few orders of magnitude less than the Planck energy scale and is approximately of order $M_P/100 \sim 10^{16}\text{GeV}$ according to the observational results, while we expect that some period of time in the inflationary era takes place in the energy scale of order $M_P \sim 10^{18}\text{GeV}$ [21]. In most of the conventional inflationary models, this situation is impossible because inflation begins from the energy scale of order $M_P/100 \sim 10^{16}\text{GeV}$ and remains in this energy such that the horizon exit occurs in this energy scale. But in our model, inflation begins from the energy scale of order M_P and then it converges rapidly to the energy scale of order $M_P/100$ at which the slow-roll behavior occurs so that the horizon exit takes place in this energy scale. We can see this fact in Fig. 3 that we have used Eq. (26) to plot the evolution of inflationary potential versus dimensionless time from the initial time inflation until the time of the sound horizon exit determined as $\bar{t}_{*s} = 1.6 \times 10^6$ or equivalently $t_{*s} = 4.3 \times 10^{-37}\text{sec}$ from Eq. (36).

III. SOLVING THE END OF INTERMEDIATE INFLATION PROBLEM

Although we showed that the intermediate inflation in a non-canonical setting can be consistent with the observational results, it suffers from a problem known as the “graceful exit” problem in which inflation never ends. To resolve this central drawback of the intermediate inflation, following [34], we assume that the potential responsible for the intermediate inflation is indeed an approximation of a more general potential which has a minimum in its shape and can provide a graceful exit for our inflationary model. As we have already seen in section II, the inverse power law potential $V = V_0\phi^{-s}$ gives rise to the intermediate inflation in the non-canonical framework (13). A graceful exit from inflation for our non-canonical intermediate inflationary model can be provided by the following modification to the potential $V = V_0\phi^{-s}$ as

$$V(\phi) = V_0 \left[\left(\frac{\phi}{M_P} \right)^{-s/2} - \left(\frac{\phi}{M_P} \right)^{s/2} \right]^2, \quad (59)$$

where s and V_0 are still given by Eqs. (29) and (30), respectively. We take $\alpha = 16$, $A = 4$, $f = 255/1000$ and $a_i = 5.6 \times 10^{-121}$ as determined in the previous section. The plot of

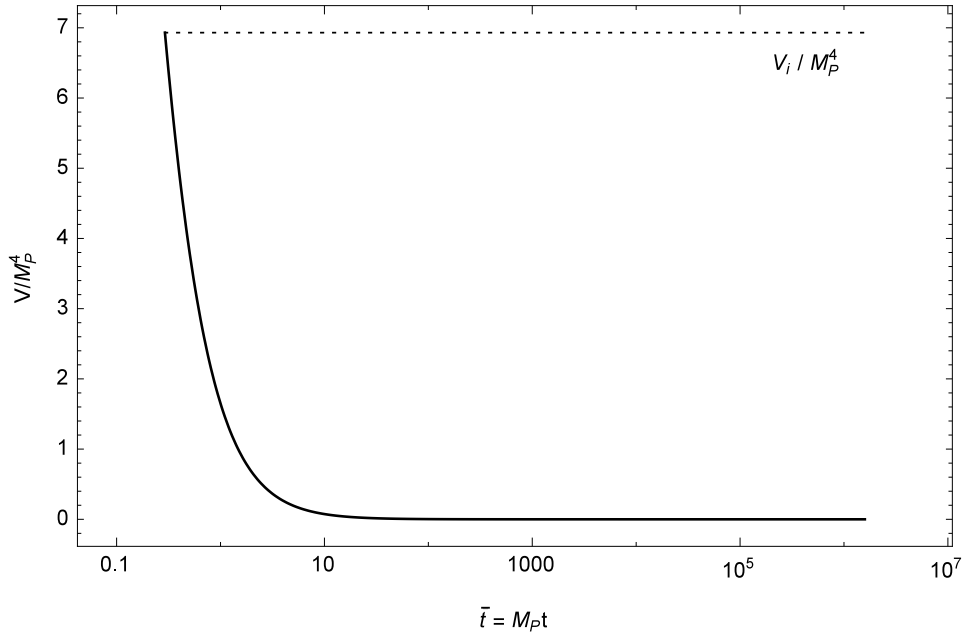


FIG. 3: Evolution of the inflationary potential versus the dimensionless time $\bar{t} = M_P t$ from the beginning of inflation until the time of sound horizon exit. The solid line corresponds to the inflationary potential (26) and the dotted line specifies the potential energy at the start of inflation.

the modified inflationary potential (59) has been shown in Fig. 4. The left branch of this potential ($\phi < M_P$) leads to the non-canonical intermediate inflation with $V \propto \phi^{-s}$, while the right branch ($\phi > M_P$) corresponds to a chaotic inflation with the potential $V \propto \phi^s$ which has already been studied in [33]. The potential (59) has a minimum at $\phi = M_P$ where $V(\phi) = 0$ and the scalar field oscillations around this minimum can provide a reheating process for the universe to transit into the radiation dominated era [9]. We will show later that the modification (59) to the potential $V = V_0 \phi^{-s}$ does not change considerably the nature of the intermediate inflation (21) until the time of horizon exit and consequently does not influence significantly on the results obtained for the inflationary observables in the previous section.

In what follows, we proceed to study the inflation from the left branch of the potential (59) specified by $\phi < M_P$. We substitute Eqs. (14) and (15) into the equation of state parameter (4) and then evaluate the obtained result at $\phi = M_P$ where the potential (59)

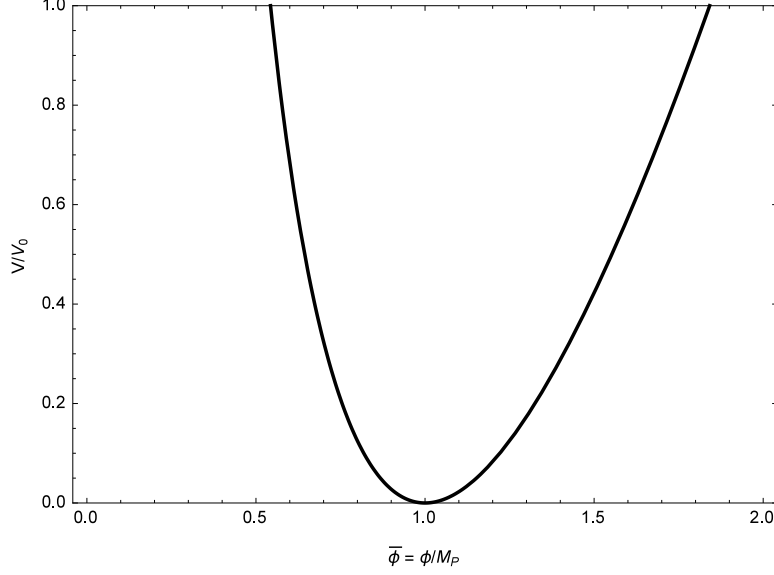


FIG. 4: The modified inflationary potential (59) versus the normalized scalar field $\bar{\phi} = \phi/M_P$ for $\alpha = 16$ and $f = 255/1000$. The left branch of the potential ($\phi < M_P$) leads to the non-canonical intermediate inflation, while the right branch ($\phi > M_P$) corresponds to a chaotic inflation.

vanishes. Therefore, we find

$$\omega_\phi = \frac{1}{2\alpha - 1} = \frac{1}{31}, \quad (60)$$

which shows that $\omega_\phi > -1/3$ and consequently inflation has ended.

To determine the end point of inflation, first we simplify the first potential slow-roll parameter (17) for the potential (59) and reach

$$\varepsilon_V = \frac{0.7935 \bar{M}^{\frac{1020}{2017}} \left(\bar{\phi}^{\frac{9536}{6051}} + 1 \right)^{\frac{32}{31}}}{\bar{\phi}^{\frac{20704}{6051}} \left(\bar{\phi}^{\frac{9536}{6051}} - 1 \right)^2}, \quad (61)$$

where $\bar{\phi} = \phi/M_P$ is the normalized scalar field. From Eq. (61), we conclude that ε_V begins from 1 at the start of inflation and then it decreases and reaches the values of $\varepsilon_V \ll 1$. Subsequently, it increases and again it approaches to 1 at the end of inflation. Therefore, the relation $\varepsilon_V = 1$ leads to two solutions corresponding to the start and end of inflation.

If we use the potential (59) in Eq. (31), we find the scalar power spectrum as

$$\mathcal{P}_s = \frac{0.1501 \bar{M}^{\frac{4940}{2017}} \left(\bar{\phi}^{\frac{9536}{6051}} - 1 \right)^4}{\bar{\phi}^{\frac{7904}{6051}} \left(\bar{\phi}^{\frac{9536}{6051}} + 1 \right)^{\frac{32}{31}}}. \quad (62)$$

To obtain the value of normalized scalar field at the horizon exit, $\bar{\phi}_*$, we should solve

$$N_* = 60 = - \int_{\bar{\phi}_e}^{\bar{\phi}_*} \frac{H}{\dot{\phi}} (M_P d\bar{\phi}), \quad (63)$$

where we have used Eq. (12). In order to compute the above integration numerically, we use

$$H = 1.300 \bar{M}^{\frac{2980}{2017}} \bar{\phi}^{-\frac{4768}{6051}} \left(1 - \bar{\phi}^{\frac{9536}{6051}}\right) M_P, \quad (64)$$

that results from Eq. (19). Also, for $\dot{\phi}$ in Eq. (63), we use

$$\dot{\phi} = 1.309 \bar{M}^{\frac{4000}{2017}} \bar{\phi}^{-\frac{349}{6051}} \left(\bar{\phi}^{\frac{9536}{6051}} + 1\right)^{\frac{1}{31}} M_P^2, \quad (65)$$

that arises from Eq. (20). Now, we set $\varepsilon_V(\bar{\phi}_e) = 1$ in Eq. (61) and fix the power spectrum (62) as $\mathcal{P}_s(\bar{\phi}_*) = 2.207 \times 10^{-9}$ according to Planck 2015 TT,TE,EE+lowP data [23]. Then, we solve the resulting equations together with Eq. (63) simultaneously by a numerical method. In this way, we find $\bar{M} = 2.6 \times 10^{-4}$, $\bar{\phi}_e = 0.8994$ and $\bar{\phi}_* = 0.1539$. With these results in hand, from Eq. (30) we obtain $V_0 = 1.3 \times 10^{-10} M_P^4$.

Now, we are in a position to show that the modification of the inflationary potential in the form of (59) does not alter significantly the nature of the intermediate inflation until the time of horizon exit. For this purpose, we solve Eqs. (64) and (65) simultaneously in a numerical approach and find time evolutions of the scale factor $a(\bar{t})$ and the scalar field $\bar{\phi}(\bar{t})$. Here $\bar{t} \equiv M_P t$ is dimensionless time. Then, with the help of the scale factor, we can set the first slow-roll parameter (9) equal to 1 and find the initial and end time of inflation as $\bar{t}_i = 0.29$ and $\bar{t}_e = 8.2 \times 10^6$, respectively. Therefore, our model predicts the initial and end time of inflation as $t_i = 7.9 \times 10^{-44}$ sec and $t_e = 2.2 \times 10^{-36}$ sec, respectively. Also, using the obtained value for ϕ_* , our model gives the time of horizon exit as $\bar{t}_* = 1.3 \times 10^6$ or equivalently $t_* = 3.5 \times 10^{-37}$ sec. In Fig. 5, we plot the evolution of scale factor versus dimensionless time from the beginning until the end of inflation. Figure 5 clears that the nature of intermediate scale factor does not change considerably until the time of horizon exit. We can also see this fact in Fig. 6 that shows the variations of the first slow-roll parameter (9) versus dimensionless time. We see in this figure that the first slow-roll parameter (9) corresponding to the modified potential (59) is very similar to the one corresponding to the intermediate scale factor (21), from the start of inflation to the time of horizon exit. Therefore, we conclude that the considered modification for the inflationary potential does

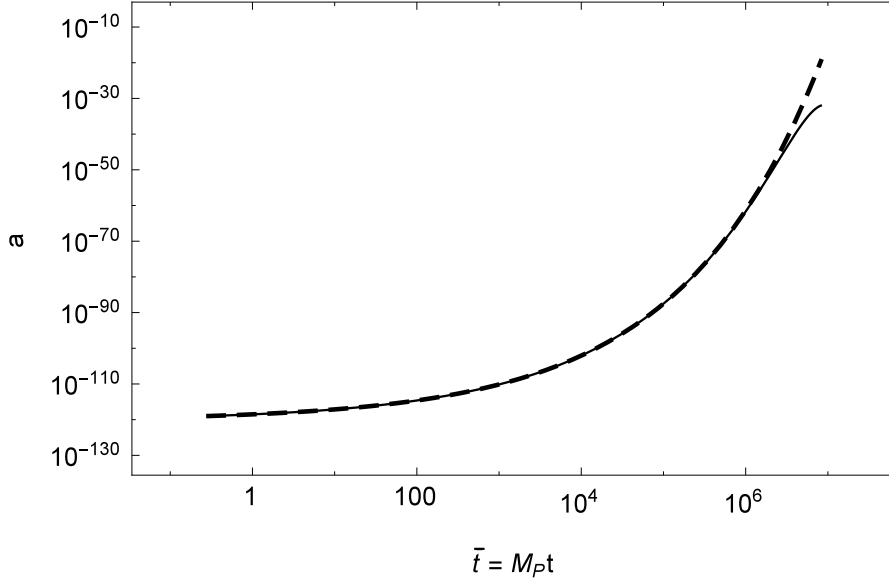


FIG. 5: Evolution of the scale factor versus the dimensionless time $\bar{t} = M_P t$ from the beginning until the end of inflation. The dashed line shows the intermediate scale factor resulting from the original inflationary potential (28) while the solid line shows the scale factor corresponding to the modified inflationary potential (59).

not disturb the predictions made for the inflationary observables in the previous section because these quantities are evaluated at the horizon exit.

IV. CONCLUSIONS

Here, we investigated the intermediate inflation characterized by the scale factor $a(t) = a_i \exp \left[A(M_P t)^f \right]$ where $A > 0$ and $0 < f < 1$ in a non-canonical framework with a power-like Lagrangian $\mathcal{L}(X, \phi) = X \left(\frac{X}{M^4} \right)^{\alpha-1} - V(\phi)$. This Lagrangian is a natural generalization of the standard canonical one. We showed that in our non-canonical framework, the intermediate inflation is driven by the inverse power law potential $V = V_0 \phi^{-s}$. Having the inflationary potential in hand, we turned to check the viability of our model in light of the observational results from Planck 2015. We first plot the $r - n_s$ diagram for our model and showed that although the standard canonical model ($\alpha = 1$) is not favored in light of the Planck 2015 observational results, our non-canonical model of intermediate inflation can be compatible with Planck 2015 results if we choose the parameter α sufficiently large. Setting $\alpha = 16$ and

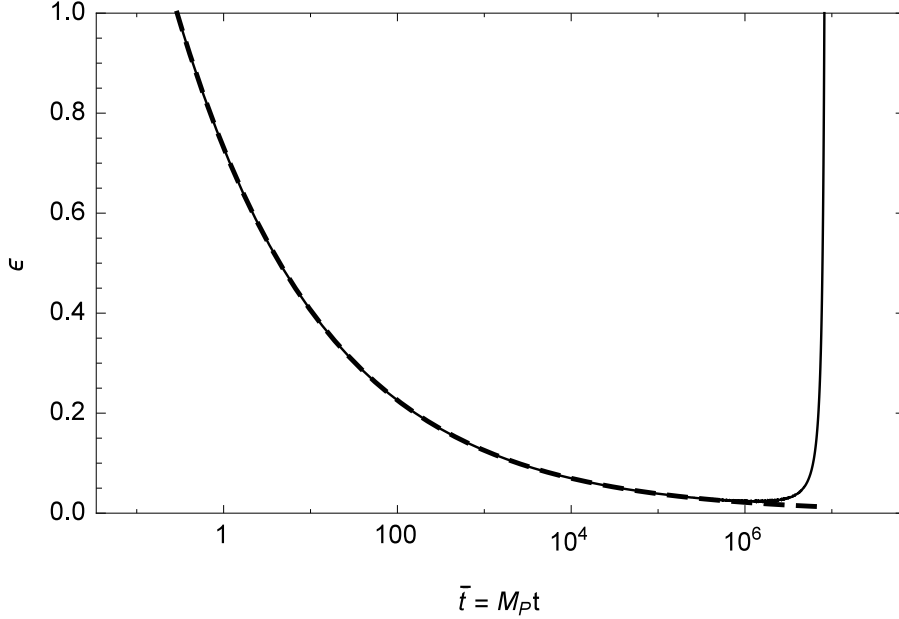


FIG. 6: Evolution of the first slow-roll parameter (9) versus the dimensionless time $\bar{t} = M_P t$ from the beginning until the end of inflation. The dashed line indicates the first slow-roll parameter (9) corresponding to the original inflationary potential (28) and the solid line shows the one relating to the modified inflationary potential (59).

$A = 4$, we also plotted the $dn_s/d \ln k$ versus n_s and showed that the prediction of our model is in agreement with Planck 2015 results. Choosing $f = 255/1000$, we got $s = 9536/6051$ and $V_0 = 5.067 \bar{M}^{5960/2017} M_P^4$. We also chose the pivot scale as $k_0 = 0.05 \text{Mpc}^{-1}$ and fixed $\mathcal{P}_s(k_0) = 2.207 \times 10^{-9}$ from Planck 2015 TT,TE,EE+lowP data combination [23] and determined $a_i = 5.6 \times 10^{-121}$. After determining the parameters of our model, we estimated the scalar spectral index, the tensor-to-scalar ratio and the running of the scalar spectral index as $n_s = 0.9676$, $r = 0.052$ and $dn_s/d \ln k = 0.0002$, respectively. These are inside the range with 68% CL predicted by Planck 2015 TT,TE,EE+lowP data [23]. Furthermore, we obtained the tensor spectral index as $n_t = -0.039$ that can be checked by the increasingly precise measurements in the future. This prediction for n_t satisfies the constraint imposed by the consistency relation together with the upper bound on the tensor-to-scalar ratio from Planck 2015 TT,TE,EE+lowP data [23]. In addition, our model predicts the equilateral non-Gaussianity as $f_{\text{NL}}^{\text{equil}} = -8.5$ which is in agreement with Planck 2015 results at 68% CL [23].

Subsequently, we obtained the energy scale at the start of inflation as $V_i^{1/4} = 1.6M_P \sim 10^{18}\text{GeV}$. Therefore, in our model, the inflation can initiate from the energy scale of order of the Planck energy scale and it rapidly converges to the energy scale of order $M_P/100 \sim 10^{16}\text{GeV}$ that we expect from the observational results. In this way, we could address to one of the mysteries of the inflation theory implying that we expect a period of time in the inflationary era to occur in the Planck energy scale but the observational results show that the energy density of the universe at horizon exit is a few orders of magnitude less than the Planck energy scale. We can't resolve this problem in most of the conventional inflationary models because in them, inflation begins from the energy scale of order $M_P/100 \sim 10^{16}\text{GeV}$ and remains in this energy scale such that the horizon exit takes place in this energy scale.

We also examined an idea to resolve the graceful exit problem of intermediate inflation in non-canonical framework. In order for inflation to end, we considered a modification to the inflationary potential and showed that this modification does not disturb significantly the nature of the intermediate inflation until the time of horizon exit. Therefore, the obtained results for the inflationary observables does not change considerably. Using the modified inflationary potential, we computed $\bar{M} = 2.6 \times 10^{-4}$ that leads to $V_0 = 1.3 \times 10^{-10}M_P^4$. We found that our non-canonical intermediate inflationary model gives the start time of inflation, the time of horizon exit and the end time of inflation as $t_i = 7.9 \times 10^{-44}\text{sec}$, $t_* = 3.5 \times 10^{-37}\text{sec}$ and $t_e = 2.2 \times 10^{-36}\text{sec}$, respectively.

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